Viscosity and Its Variation

Don C. Warrington
University of Tennessee at Chattanooga
Departments of Civil and Mechanical Engineering

Viscosity is an important quantity in fluid mechanics. This monograph is an overview of its definition, how it varies in some fluids, how to measure it, the units of viscosity, and dynamic vs. kinematic viscosity.

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Introduction

The definition of a fluid is "a substance that deforms continuously when subjected to a shear stress, no matter how small that shear stress may be." (Streeter (1966)) In Mechanics of Materials, we learn that materials can be stress and deformed in one of two ways: with normal stresses and shear (tangential) stresses. With fluids at rest (static case,) there are only normal stresses, or what we normally call pressures. For these fluids, there are no shear stresses. But when fluids move, we can have shear stresses. These stresses are referred to as viscous stresses. Viscosity "is the fluid property that causes shear stresses in a moving fluid..." In this monograph we will discuss this, with an emphasis on experimental application.

Fluids which have no viscosity are referred to as "inviscid." Inviscid fluids exist only in a computer or in theory. All real fluids have some viscosity. In some cases (such as air) the viscosity can be neglected for some analytical purposes, and this is done frequently in computational fluid dynamics (CFD) using, for example, the Euler Equations. But all of these fluids have viscosity, be it ever so small. As a practical matter, fluids in motion cannot be understood without considering viscosity. This is then the task to which we set ourselves.

Definition of Viscosity

Viscosity is generally defined using Equation 1:

$$\tau = \mu \frac{du}{dy} \quad (1)$$

where

- τ =shear stress
- μ =dynamic viscosity
- u =fluid velocity
- y =distance
- $\frac{du}{dy}$ =shear gradient

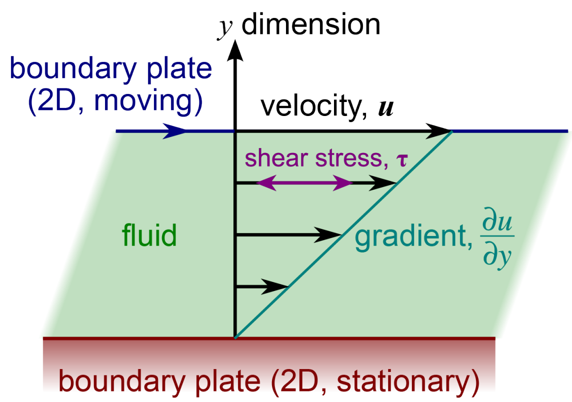

Figure 1. Fluid Shear Between Two Parallel Plates (from New World Encyclopedia)

To get some understanding of what Equation 1 "means," consider the situation depicted in Figure 1.

We have two plates, one stationary and one moving with a constant velocity. We assume that the fluid particles next to the plates assume the velocity of the plates themselves. This is also a basis for boundary layer theory. We thus have fluid particles between the plate whose velocity varies, reasonably enough, between that of the stationary plate and the moving plate. We will assume that these velocities vary linearly between the two plates, and that the shear stress in the fluid generated by this variation is directly proportional to that linear variation, which is the gradient of the shear stress. The constant of proportionality μ is referred to as the dynamic viscosity, or more generally the viscosity of the fluids.

Situations such as depicted in Figure 1 have been observed experimentally. Fluids which have a uniform gradient, and for which μ =constant, are referred to as Newtonian fluids. Many fluids, including most common liquid lubricants, water and air itself are Newtonian fluids. It's also worth noting that the flow regime in Figure 1, where the gradient is linear or at least smooth, is a laminar flow regime, and this one specifically is referred to as Couette flow. This will be of more importance with deeper study of fluid mechanics.

Variation of Viscosity

We said that μ =constant; we know from experience, however, that viscosity varies with, for example, temperature. Oil, for example, flows more freely (viscosity is lower) at higher temperature. One of the challenges in designing motor oil is for it to maintain adequate viscosity to lubricate the running surfaces at a wide variety of temperatures. We should thus modify Equation 1 to the following:

$$\tau = \mu(T)\frac{du}{dy} \tag{2}$$

When doing any measurement of viscosity, it is necessary to report the temperature under which the experiment is being run. If a fluid is tested for actual use, a number of temperatures probably need to be analyzed.

But what happens when μ varies with the shear stress itself? Well, we should rewrite Equation 2 to this:

$$\tau = \mu\left(T, \frac{du}{dy}\right)\frac{du}{dy} + \tau_0 \tag{3}$$

To get a better idea of what this really means, consider Figure 2.

The lower plot (Shear Stress vs. Rate of Shear) is basically a plot of Equation 3. The upper plot (Viscosity vs. Rate of Shear) is basically the first derivative of that same equation, or

$$\frac{d\tau}{d\left({}^{du}/_{dy}\right)} = \mu \tag{4}$$

The upper plot is a little easier to understand so let us consider it in detail. Some of this discussion is taken from Gartmann (1970).

For Newtonian fluids, by definition the viscosity is constant with the rate of shear $\frac{du}{dy}$, and so the plot is a straight horizontal line. Note also, however, that the plot for plastic materials also has a constant viscosity with rate of shear. The difference, as is obvious in the lower plot, is that $\tau_o > 0$. Plastic (or Bingham Plastic) materials are those which act like solids (with shear resistance) up to a point, and then act as a liquid. An example of this is ketchup; when you attempt to beat it out of the bottle, it at first goes nowhere, but eventually comes out in a glop. If this happens on a date, and the glop gets on your date, you can sheepishly look at your evening companion and say, “Bingham Plastic.” (If your date is an engineer, you just might get away with this, otherwise...)

The other two are mirror images of each other. Pseudoplastic fluids are those which have a high viscosity at low shear rates but lower with higher shear rates. The most familiar example of this is human blood, whose viscosity actually changes during each beat of the heart, going down during peak velocity (systole) and rising during minimum velocity (diastole.) Elevated viscosity of blood has been linked with virtually all coronary disease because the higher viscosity results in high shear stresses along the arterial walls. Other pseudoplastic materials include water-based fluids and resinous materials.

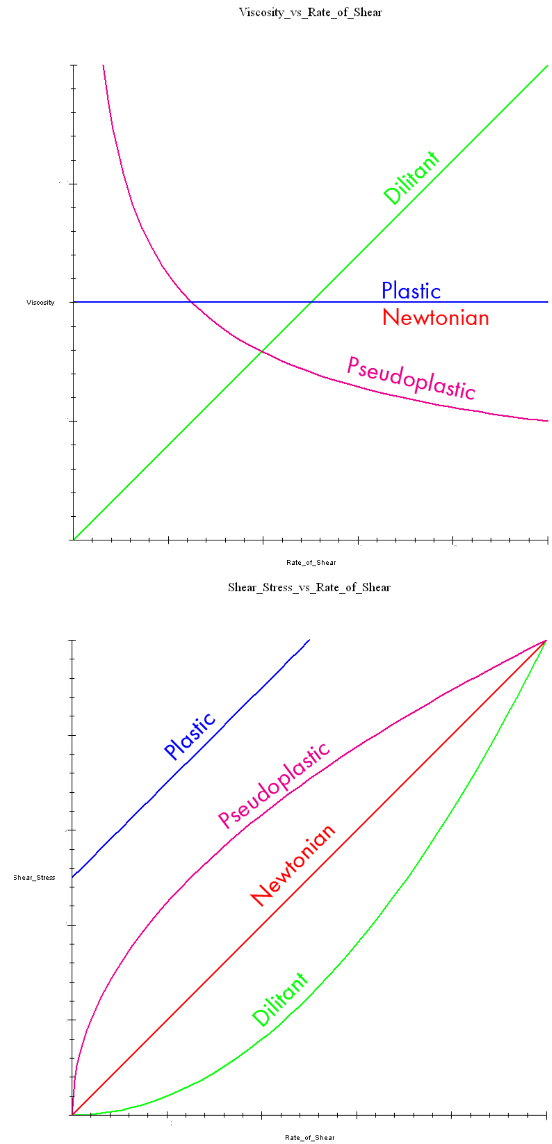

Figure 2. Variation of Viscosity (Top) and Shear Stress (Bottom) with Rate of Shear $\frac{du}{dy}$

At the other end of the spectrum are dilitant fluids, which have low viscosity at low shear rates but whose viscosity increases with the shear rate. In some cases they solidify at a high shear rate. Examples of these include paints, printing inks and some starches.

It should be noted that, although the viscosity of dilitant fluids looks linear in Figure 2 while the pseudoplastic does not, this is not necessarily the case. They each can be linear or non-linear; it is the increasing or decreasing trend that defines whether a fluid is pseudoplastic or dilitant.

Finally, there are other factors other than temperature and rate of shear that affect viscosity. Some fluids, for example, change their viscosity with time, and that change can be accelerated by the environment they are in. There are two types of fluids whose viscosity changes with time. Fluids which, under shear, decrease their viscosity with time are referred to as thixotropic fluids, while those which increase their viscosity with time are rheopectic. In some cases the change in viscosity is reversible and sometimes it is not.

Measurement of Viscosity

Quantification is at the heart of engineering. Unless a property can be quantified, it cannot be used in engineering practice. Viscosity is no exception, and its measurement affects the way it is reported.

For many years the most common way to measure viscosity was to use a Saybolt viscometer. The concept was simple: measure the time it took for 60 cc of fluid to pass through a standard orifice (Mott (1994).) There was more than one standard orifice, but the time required to pass through the most commonly used one is referred to as Saybolt Universal Seconds (SUS) or Saybolt Seconds Universal (SSU.) It was a simple test to run, and for Newtonian fluids (whose viscosity is constant with shear) gave reasonable results. However, because it cannot be used to check if a fluid is Newtonian or not, and for other reasons, it has passed out of currency. Nevertheless you will find viscosities quoted in SSU or SUS, thus the need to be aware of it.

It would make sense to measure viscosity in such a way that the condition depicted in Figure 1 is replicated in the test. Using a pair of flat plates, however, can be tricky. An easier way to accomplish this is to use a rotating drum viscometer, such as is shown in Figure 3.

The concept is that weights (such as those lying around the table) are placed on the weight deck, which in turn pulls on the string. The string, through the winch at the top and the gear mechanism, applies a torque to the rotor, which in this case is immersed in a milky fluid. Surrounding rotor and fluid is the fixed cylinder. As different weights are applied, different torques are applied to the fluid, which in turn applies differing shear forces and stresses. By applying a series of weights, the viscosity can be determined for a variety of shear stresses, and thus not only can the viscosity be determined but whether the fluid is Newtonian or not–and if not, how it is non-Newtonian.

If the conditions of Figure 1 apply, the shear stress can be given by the equation

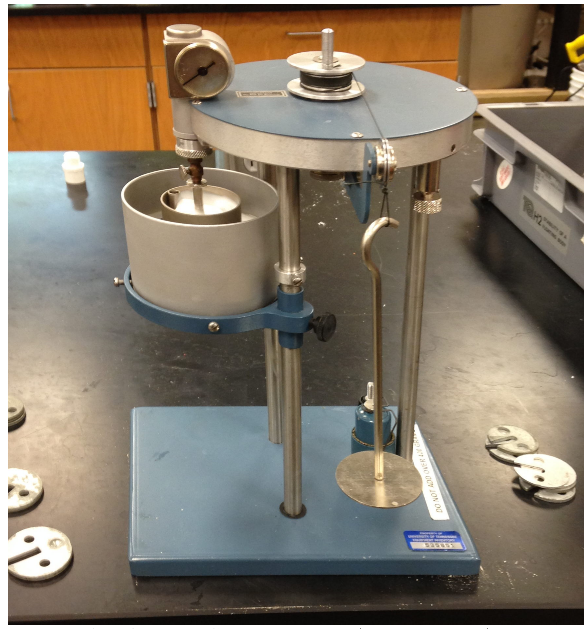

Figure 3. Thomas-Stormer Rotating Drum Viscometer

$$\tau = \mu k \omega \tag{5}$$

where

- k =constant based on the geometry of the viscometer
- ω =angular velocity of the rotor

The torque exerted on the fluid by the weight mechanism is

$$T = \tau A r_r \tag{6}$$

where

- T =torque exerted on the fluid
- A =whetted area of drum
- r_r =radius of drum

Since we are measuring a dynamic quantity, we can equate the power required to rotate the rotor in the fluid with the power gravity exerts on the falling weight, or

$$T\omega = Wu \tag{7}$$

where

- W =applied weight

It is possible at this point to combine Equations 5, 6 and 7 and, considering the geometry of the system and the properties of the mechanism, to come up with an equation to relate the viscosity of the fluid to the weight and the angular velocity. Generally this relationship is determined for a given viscometer with both theoretical and experimental considerations, and it is in the following form:

$$\mu = C\frac{W}{\omega} \tag{8}$$

The simplest way to measure angular velocity is to measure the time to achieve a certain number of rotations. In practice, Equation 8 can be applied using

$$\mu = Kt \tag{9}$$

where

- K =constant which includes (and thus varies with) the weight applied

Rotating drum viscometers can (and frequently do) generate their torque using an electric motor with electronic instrumentation.

By applying different weights (and thus different torques) we can determine the viscosity at a variety of shear rates. We can plot these in a similar manner to the upper plot of Figure 2 and determine whether a fluid is Newtonian or not and, if not, whether it is pseudoplastic or dilitant, or even if it is thixotropic or rheopectic.

Kinematic vs. Dynamic Viscosity

When we refer to viscosity, we're generally talking about "dynamic viscosity," which is defined in Equation 1. But we can also use a quantity referred to as "kinematic viscosity," which is defined as

$$\nu = \frac{\mu}{\rho} \tag{10}$$

where

- ν =kinematic viscosity
- ρ =density of fluid

When looking up experimental or typical values of viscosity, make sure you know whether you're looking at dynamic or kinematic viscosity. Kinematic viscosity is a very useful quantity, especially for incompressible fluids whose density doesn't change a great deal. It also makes unit analysis much simpler, as we will see shortly.

Units of Viscosity

Units are one of the trickiest parts of fluid mechanics, and viscosity is no exception. The confusion is amplified both by the existence of US and SI units and also empirical measurements such as the SSU.

Examination of Equation 1 would indicate that dynamic viscosity μ would be reported in $^{lb_f-sec}/_{ft^2}$ (or $^{slug}/_{ft-sec}$) in US units and $^{N-sec}/_{m^2}$ (or $Pa-sec$) for SI units. However, it is seldom reported in either of these units. The most common unit of measure for dynamic viscosity is the poise, which is in $^{dyne-sec}/_{cm^2}$ or $^{g}/_{cm-sec}$, which is a CGS unit! The results of this for many liquids are numerically low, so we generally employ the centipoise cP, or multiply the poise by 100. For use in US units calculation, we use the conversion $47,880.1\ ^{cP-ft^2}/_{lb_f-sec}$.

The units for kinematic viscosity ν are $^{ft^2}/_{sec}$ in US units and $^{m^2}/_{sec}$ for SI units. This is pretty simple; in fact, one of the strong points of using kinematic viscosity is that you don't get lost in the force vs. mass problem. However, the unit usually employed for kinematic viscosity is the stoke, named after the British scientist G.G. Stokes, and is also a CGS unit ($^{cm^2}/_{sec}$.) As is the case with the poise, we generally use the centistoke cSt. The conversion for US units is $92,903.4\ ^{cSt-sec}/_{ft^2}$.

And for those pesky Saybolt units, if you run across the standard SSU (or SUS) the conversion is as follows (Gartmann (1970)):

$$\begin{aligned} \nu_{Centistokes} &= 0.226 \times SSU - \frac{195}{SSU}, \ SSU \le 100 \\ \nu_{Centistokes} &= 0.220 \times SSU - \frac{130}{SSU}, \ SSU > 100 \end{aligned} \tag{11}$$

Finally the dynamic and kinematic viscosities for two important materials are shown in Table 1.

Table 1
Dynamic and Kinematic Viscosity for Water and Air at 68°F and Atmospheric Pressure (after Gartmann (1970))

Fluid	μ, cP	μ, $^{lb_f-sec}/_{ft^2}$	ν, cSt	ν, $^{ft^2}/_{sec}$	ν, SSU
Air	180.8×10^{-3}	0.3369×10^{-6}	15.01	161.6×10^{-6}	
Water	1.0087	21.067×10^{-6}	1.0105	10.877×10^{-6}	30.1

Viscosity Example: Permeability Through a Porous Medium

As an example of the use of units and the relationship between dynamic and kinematic viscosities, consider the permeability of a fluid flowing through a porous (usually granular) medium. This is of special interest to civil and environmental engineers because these relationships are used to estimate the flow of water (and other contaminants) through soils. Most of this section is based on Harr (1990).

By Darcy's Law (more properly expressed as d'Arcy's Law) the flow of fluids through a granular, porous medium can be expressed by the equation

$$v = -ki \tag{12}$$

where v is the discharge velocity of the fluid, i is the hydraulic gradient, and k is the coefficient of permeability (or conductivity) of the soil. The discharge velocity is the average velocity of a fluid through a medium which essentially disregards the presence of the soil particles. The hydraulic gradient i is the change in head per unit length as the fluid passes through the soil. This relationship essentially holds as long as the flow through the soil is laminar (which is usually the case with fluid flow through soils.) It is analogous to electrical resistivity, although in this case we are using the ability of the soil to transmit the flow rather than the ability of the resistor to impede the electric current.

The coefficient of permeability is in turn given by the equation

$$k = k_0 \frac{\gamma}{\mu} \tag{13}$$

where γ is the unit weight of the fluid and μ is the dynamic viscosity of the fluid. The quantity k_0 is the physical permeability of the soil, which is a property of the soil based on the way the particles are packed together, the shape and angularity of the particles, etc. This quantity is at this point difficult to determine analytically, which is why it is generally done experimentally. The units of this are, believe it or not, the units of area.

As a side note, for most applications in soil mechanics, water is the fluid in question, and most of the experimentation to determine k is done with water. However, as other fluids (specifically environmental contaminants) are introduced into the soil, their permeability becomes of interest, and this formula allows us (at least for a first analysis) to use the results we obtained for water with other fluids, or mixtures of fluids.

We can use this to determine the units of the coefficient of permeability k. Using cgs units (commonly used to determine permeability in the laboratory,)

$$cm^2 \frac{dynes/cm^3}{dyne{-}sec/cm^2} = \frac{cm}{sec}$$

which means that the coefficient of permeability is in units of velocity! This also squares with Equation 12, as the hydraulic gradient is dimensionless.

Recovering from this discovery, if we look at Equation 13 carefully, we will see that, assuming that k_0 and γ are constant, the coefficient of permeability increases as the dynamic viscosity decreases. This makes sense if we consider the effects of shear on the fluid; as the viscosity decreases, the shear of the fluid against the particles decreases, and the ability of the fluid to flow through the soil increases.

Since, however, we made a big deal out of making mistakes with units, we should ask the question: can we simplify Equation 13 with Equation 10? The answer is yes, doing so will yield

$$k = k_0 \frac{g}{\nu} \tag{14}$$

where g is the acceleration due to gravity. We can apply a unit analysis to this, too. Again using CGS units and (conveniently) expressing the kinematic viscosity in Stokes (cm^2/sec) we examine the units again:

$$cm^2 \frac{cm/sec^2}{cm^2/sec} = \frac{cm}{sec}$$

which is again valid. As with dynamic viscosity, the permeability increases as the kinematic viscosity decreases. It also shows how much units analysis can be simplified by using kinematic viscosity.

Conclusion

Viscosity is an important property of fluids. We have looked at the definition of viscosity, how it varies with different fluids, how it is measured, and how to handle the units. With this basic knowledge, many phenomena in fluid mechanics can be quantified, which is an essential prerequisite for proper design and analysis of mechanisms with fluids in motion.

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